

Middle and Upper School Mathematics Standards & Benchmarks

1. Students will apply a wide variety of mathematical concepts, processes, and skills to solve a broad range of problems in various content areas and everyday situations

Grade 6	Grade 7	Grade 8	Grade 9 & 10	Grade 11 & 12
Explore and solve a variety of problems that have a variety of possible strategies or solutions or both, in order to build new mathematical knowledge.	Pose, explore, and solve a variety of problems that have a variety of possible strategies or solutions or both, in order to build new mathematical knowledge.	Pose, explore, and solve a variety of problems, including those that are non-routine or have a variety of possible strategies, solutions, or both in order to build new mathematical knowledge.	Explore the validity and efficiency of various problem-posing and problem-solving strategies; develop alternative strategies and generalizations as needed.	Explore the validity and efficiency of various problem-posing and problem-solving strategies; develop alternative strategies and generalizations as needed.
Apply a variety of problem-solving techniques to solve problems (for example, working backward, information organizers, or solving a similar but simpler problem).	Apply an increasing variety of problem-solving techniques to solve problems.	Develop, modify, and apply an increasing variety of problem-solving techniques to solve problems (for example, working backward, information organizers, or solving a similar but simpler problem).	Monitor progress toward solutions.	Monitor progress toward solutions.
Try various problem-solving approaches before selecting and using a strategy.	Try various problem-solving approaches before selecting and using a strategy.	Try various problem-solving approaches before selecting and using a strategy and reflect on different strategies used when a task is complete.	Generalize strategies and reflect on their proficiency and merit.	Generalize strategies and reflect on their proficiency and merit.

2. Students will apply mathematical reasoning skills to make sense of, investigate, evaluate, justify, and connect approaches and solutions to situations in mathematics and in other disciplines.

Grade 6	Grade 7	Grade 8	Grade 9 & 10	Grade 11 & 12
		Make and investigate mathematical conjectures.	Construct, follow, and evaluate arguments by using reasoning and logic.	Construct, follow, and evaluate arguments by using reasoning and logic.
		Use "if...then" statements to construct simple valid arguments.	Use a variety of methods and proofs (e.g. using direct, indirect, informal, truth tables, and paragraphs) to validate conjectures.	Use a variety of methods and proofs (e.g. using direct, indirect, informal, truth tables, and paragraphs) to validate conjectures.
	Use inductive and deductive reasoning appropriately.	Use inductive and deductive reasoning appropriately.	Relate procedures in one representation of a problem to procedures in an equivalent representation.	
Apply proportional and spatial thinking.	Apply proportional and spatial thinking.	Apply proportional and spatial thinking.	Use the connections among mathematical topics to develop multiple approaches to problems.	
Begin to develop mathematical arguments and informal proofs.	Develop mathematical arguments and informal proofs.	Develop and evaluated mathematical arguments and informal proofs.	Demonstrate how graphs can be used to model real-world situations and to determine solutions to numerous problems involving algebraic functions.	
Begin to identify and use connections between various mathematical topics to build upon existing knowledge.	Identify and use connections between various mathematical topics to build upon existing knowledge.	Identify and use connections between various mathematical topics to build upon existing knowledge.		Relate procedures in one representation of a problem to procedures in an equivalent representation.
Use mathematical ideas from one area of mathematics (e.g., an equation or formula from algebra) to explain an idea from another area of mathematics (e.g., the area of a triangle).	Use mathematical ideas from one area of mathematics (e.g., an equation or formula from algebra) to solve problems from another area of mathematics (e.g., the area of a triangle).	Use mathematical ideas from one area of mathematics (e.g. and equation or formula from algebra) to explain an idea from another area of mathematics (e.g. the area of a triangle) and demonstrate how mathematical ideas are a coherent whole.		Use the connections among mathematical topics to develop multiple approaches to problems.
Apply mathematical skills and processes to other disciplines and to everyday situations.	Apply mathematical skills and processes to other disciplines and to everyday situations.	Apply mathematical skills and processes to other disciplines and everyday situations.		Demonstrate how graphs can be used to model real-world situations and to determine solutions to numerous problems involving algebraic functions.

3. Students will understand mathematical information presented and obtained in a variety of ways and will accurately and clearly present and justify mathematical ideas in diverse formats.

Grade 6	Grade 7	Grade 8	Grade 9 & 10	Grade 11 & 12
Demonstrate a variety of mathematical representations and use them appropriately.	Demonstrate and use a repertoire of mathematical representations appropriately.	Demonstrate a repertoire of mathematical representations and use them purposefully, flexibly, and appropriately.	Formulate questions, conjectures, and generalizations, about data, information, and problem situations.	Formulate questions, conjectures, and generalizations, about data, information, and problem situations.
Build on mathematical language, notation, and symbols as a precise means of expressing problem situations and mathematical ideas.	Build on mathematical language, notation, and symbols as a precise means of expressing problem situations and mathematical ideas.	Use mathematical language, notation, and symbols as a precise means of expressing problem situations and mathematical ideas.	Present complete and convincing arguments and justifications adapted to be effective for various audiences.	Present complete and convincing arguments and justifications adapted to be effective for various audiences.
Consider mathematical arguments and conclusions presented by others.	Consider mathematical arguments and conclusions presented by others.	Analyze, evaluate, and explain mathematical arguments and conclusions presented by others.	Use technology (e.g. graphic calculators, spreadsheets, and graphing programs) to present information and ideas	Use technology (e.g. graphic calculators, spreadsheets, and graphing programs) to present information and ideas
			Use properties, models, known facts, and relationships to explain and defend thinking.	Use properties, models, known facts, and relationships to explain and defend thinking.

4. Students will select and use a wide variety of tools and technology to support and validate mathematical results, when appropriate.

Grade 6	Grade 7	Grade 8	Grade 9 and 10	Grade 11&12
Use a variety of technologies, including computers and calculators and recognize when these are not appropriate.	Use a variety of technologies, including computers and calculators and recognize when these are not appropriate.	Use a variety of technologies, including computers, scientific calculators, graphing calculators, science probes, and digital cameras to evaluate and validate problem solutions.	Use graphic calculators and computer software effectively and efficiently to define and solve various types of problems.	Use graphic calculators and computer software effectively and efficiently to define and solve various types of problems.
	Recognize situations when calculator use yields misleading or inappropriate results.	Recognize situations when a calculator use is not appropriate (e.g. when solving a simple quadratic equation which could be factored) or when it yields misleading results (e.g. when a non-linear curve appears linear).		

5. Students will understand and apply numbers, ways of representing numbers, relationships among numbers, and number systems.

Grade 6	Grade 7	Grade 8	Grade 9 & 10	Grade 11 & 12
	Demonstrate a conceptual understanding of irrational numbers and be able to solve problems involving rational numbers (e.g., area of circles, working with radicals).	Demonstrate a conceptual understanding of irrational numbers and be able to solve problems involving rational numbers (e.g. areas of circles and working with radicals).	Demonstrate a conceptual understanding of irrational numbers and be able to solve problems involving rational numbers (e.g. areas of circles and working with radicals).	
Describe how percent, ratio, and proportion apply to some mathematical situations	Describe how percent, ratio, and proportion apply to some mathematical situations.	Describe how percent, ratio, and proportion apply to mathematical situations (such as rate and similar triangles).	Solve problems involving percent; ratio, and proportion apply to mathematical situations (such as rate, similar triangles).	
Recognize and apply multiple representations of rational numbers, exponents, compare these numbers, find their approximate locations on a number line.	Recognize and apply multiple representations of rational and irrational numbers, exponents, find their approximate locations on a number line, and choose appropriate forms of these numbers.	Recognize and apply multiple representations of rational and irrational numbers, exponents, absolute values, and scientific notation; compare these numbers accurately, and find their approximate locations on a number line, and choose appropriate forms of these numbers.	Recognize properties of rational and irrational numbers, exponents, absolute values, and scientific notation.	
Use the associative and commutative properties of addition and multiplication, and the distributive property of multiplication over addition to simplify computations.	Demonstrate an understanding of numbers that represent large and small values, including the use of benchmarks to comprehend their magnitude.	Demonstrate an understanding of numbers that represent large and small values, including the use of benchmarks to comprehend their magnitude; and recognize, understand, and appropriately use various representations for large numbers.	Understands the concept of infinity.	Understands the concept of infinity.
Represent and explain the effect of operations on positive and negative numbers.	Represent and explain the effect of operations on positive and negative numbers.	Represent and explain the effect of operations on positive and negative numbers.	Explain the effect of operations on measurements (e.g. the imprecise nature of measurement is amplified with multiplication).	Explain the effect of operations on measurements (e.g. the imprecise nature of measurement is amplified with multiplication).
Add, subtract, multiply, and divide fractions, decimals, integers, and calculate simple percents.	Add, subtract, multiply, and divide fractions, decimals, percents, integers, and non-negative whole number exponents.	Add, subtract, multiply, and divide fractions, decimals, percents, integers, and non-negative whole number exponents.	Add, subtract, multiply, and divide fractions, decimals, percents, integers, integer and rational exponents.	
Understand and use the inverse relationships of addition and subtraction, multiplication and division to simplify computations and solve problems.	Understand and use the inverse relationships of addition and subtraction, multiplication and division, number squaring and square roots to simplify computations and solve problems.	Understand and use the inverse relationships of addition and subtraction, multiplication and division, number squaring and square roots to simplify computations and solve problems.		
Use factors, multiples, and prime factorisation to solve problems.	Use factors, multiples, and prime factorisation to solve problems.	Recognize and use the associative and commutative properties of addition and multiplication, and distributive properties of multiplication over addition to simplify computations with rational numbers.		
Model and connect physical, verbal, and symbolic representations of real numbers.		Model and connect physical, verbal, and symbolic representations of real numbers by hand and/or with a graphing calculator.		

6. Students will estimate, compute, and assess reasonableness of solutions.

Grade 6	Grade 7	Grade 8	Grade 9 & 10	Grade 11 & 12
		Solve problems using rates and understand rate as a unit of measurement.	Apply indirect methods, such as ratios and trigonometry, to find missing dimensions.	Manipulate algebraic procedures with real and complex numbers.
Use algorithms for computing with fractions, decimals, percents, and integers and compute them efficiently and accurately with and without a calculator.	Use algorithms for computing with fractions, decimals, percents, and integers and compute them efficiently and accurately with and without a calculator.	Use algorithms for computing with fractions, decimals, percents, and integers and compute them efficiently and accurately with and without a calculator.	Manipulate algebraic procedures with real numbers and apply exponents to solve practical problems.	Apply factorials, exponents, and matrices to solve practical problems. Compute permutations and combinations.
		Use multi-step computational procedures with rational and irrational numbers.	Estimate algebraic solutions on a graphing utility.	Estimate algebraic solutions and asymptotic behaviours of graphs on a graphing utility.
Use multi-step computational procedures with rational numbers.	Use multi-step computational procedures with rational numbers.	Estimate the value of irrational numbers.	Assess the error resulting from estimation and rounding by using metric units.	Assess the error resulting from estimation and rounding by using metric units.
Develop, analyse, and explain methods to solve problems involving proportions and percents (such as scaling, finding equivalent ratios).	Develop, analyse, and explain methods to solve problems involving proportions and percents (such as scaling, finding equivalent ratios).	Develop, analyse, and explain methods to solve problems involving proportions and percents (such as scaling and finding equivalent ratios).		
Compute circumference, area, surface area, and volume of some geometric figures (e.g. cubes, cuboids).	Compute circumference, area, surface area, and volume of geometric figures; find missing dimensions of right triangles using the Pythagorean theorem.	Compute circumference, area, surface area, and volume of geometric figures; find missing dimensions of right triangles using the Pythagorean theorem.		
	Estimate the value of tips, discounts, and taxes using host country and other currencies.	Estimate the value of tips, discounts, and taxes using host country and other currencies.		
Apply the rules of divisibility and prime factorisation.	Explain and apply the rules of divisibility, square numbers, and prime factorisation.	Explain and apply the rules of divisibility, square numbers, prime factorisation, and the properties of zero with the order of operations.		
		Determine what a reasonable degree of accuracy would be in particular situations (e.g. great precision is required in scientific experiments, but much less is required in estimating a grocery bill).	Determine when to use exact value solutions and distinguish between exact value and approximate values.	Determine when to use exact value solutions and distinguish between exact values and approximate values.

7. Students will estimate and measure to a required degree of accuracy and precision by selecting and using appropriate units, tools, and technologies.

Grade 6	Grade 7	Grade 8	Grade 9 & 10	Grade 11&12
Estimate and measure angles and use formulas to find perimeter, area, and circumference of plane figures and the volume and surface area of cubes, cuboids.	Estimate and measure angles and use formulas to find perimeter, area, and circumference of plane figures and the volume and surface area of cylinders to a specified degree of accuracy.	Estimate and measure angles and use of formulas to find perimeter, area, and circumference of plane figures and the volume and surface area of prisms, pyramids, and cylinders to a specified degree of accuracy	.	
Select and use appropriate units and tools to measure length, area, volume, angle, and weight to appropriate levels of precision.	Select and use appropriate units and tools to measure length, area, volume, angle, and weight to appropriate levels of precision.	Select appropriate units and tools to measure length, area, volume, angle, and weight to appropriate levels of precision.	Incorporate appropriate units into all aspects of measurement problems	Incorporate appropriate units into all aspects of measurement problems.
Convert between monetary systems.	Convert measurements within and between monetary systems and within and between metric and customary systems.	Convert measurements within and between monetary systems and within and between metric and customary systems; and demonstrate an understanding of the relationship between units in metric and customary systems.		
Select and use a variety of methods and tools to construct and compare plane figures of given measures.	Select and use a variety of methods and tools to construct and compare plane figures of given measures.	Select and apply indirect methods of measurement including formulas, geometric ideas and relationships (such as congruence, similarity, and the Pythagorean theorem) and algorithms for mathematical relationships (such as scale, proportions, and rate).		Apply indirect methods, such as ratios and trigonometry to find missing dimensions.
Apply information about time zones and elapsed time to solve problems.	Solve problems involving elapsed time.	Select a variety of methods and tools to construct and compare plane figures of given measures.		Interpret various international measurement systems (e.g. Richter Scale and decibels) to describe phenomena and solve problems.
		Apply information about time zones and elapsed time to solve problems.		
			Explain the relationship among error, precision, and accuracy in measurement and including the compounding of errors in calculations.	Explain the relationship among error, precision, and accuracy in measurement and including the compounding of errors in calculations.

8.Students will use algebraic methods to represent, analyse, and solve abstract and practical mathematical situations involving patterns and functional relationships.

Grade 6	Grade 7	Grade 8	Grade 9 & 10	Grade 11&12
Analyse, create, and generalize numeric and visual patterns.	Analyse, create, and generalize numeric and visual patterns.	Analyse, create, and generalize numeric and visual patterns.	Define functions and their properties and find the inverse of a function; understand the relationship between a function and its inverse.	Define functions and their properties and find the inverse of a function and understand the relationship between a function and its inverse.
Plot points in any quadrant of the coordinate plane.	Describe relationships and graphs on the coordinate plane, with particular attention to the horizontal and vertical intercepts, points of intersection, and slope for linear relationships.	Describe relationships between symbolic expressions and graphs on the coordinate plane, with particular attention to the horizontal and vertical intercepts, point of intersection, and slope for linear relationships.	Create and solve linear and quadratic equations and inequalities.	Create and solve linear and quadratic equations and inequalities.
		Analyse functional relationships to explain how a change in one quantity results in a change in another (e.g. the relationship between length, area, and volume).	Add, subtract, multiply, divide, and simplify rational and irrational expression; and add, subtract, multiply, and divide polynomials.	Add, subtract, multiply, divide and simplify rational and irrational expressions; add, subtract, multiply, and divide polynomials.
Find the value of a variable by evaluating formulas and algebraic expressions for given values (for example, if an object has a length of 4 and an area of 28, what is the object's width?).	Find the value of a variable by evaluating formulas and algebraic expressions for given values (for example, if an object has a length of 4 and an area of 28, what is the object's width?).	Find the value of a variable by evaluating formulas and algebraic expressions for given values (e.g. if an object has a length of 4 and an area of 28, what is the objects width?).	Identify, graph, and describe the graphs of basic families of functions including linear, quadratic, and trigonometric functions.	Identify, graph, and describe the graphs of basic families of functions including linear, absolute values, quadratic, exponential, and trigonometric functions. Be able to explain why a variety of phenomena can be modelled by the same type of function.
	Rewrite formulas in terms of the missing variable (for example, "if an object has a length of 4 and an area of 28, what is the object's width?" can be expressed as $4W=28$ or as $W=28/4$).	Rewrite formulas in terms of the missing variable (e.g. if an object has a length of 4 and an area of 28, what is the objects width? Which can be expressed as $4W=28$ or as $W=28/4$).	Solve systems of equations and inequalities.	Solve systems of equations and inequalities.
		Create expressions, equations, and inequalities to represent problem situations and to solve problems involving linear relationships.	Formulate expressions, inequalities, systems of equations, and systems of inequalities to model problem situations.	Formulate expressions, equations, inequalities, systems of equations, systems of inequalities, and matrices to model problem situations.
	Demonstrate fluency in generating equivalent expressions for simple algebraic expressions and in solving linear equations.	Demonstrate fluency in generation equivalent expressions for simple algebraic expressions and in solving linear equations and inequalities.		Use matrices to organize and manipulate data, including matrix addition, subtraction, multiplication, and scalar multiplication.
		Demonstrate a basic understanding of the rate of change, including connections between a slope of a line and the constant rate of change and their meanings in context.		Use calculus methods to predict the behaviour of a function, determine optimal conditions for a given situation e.g. in economics or engineering; solve problems involving rates of change, solve differential equations
				Use discrete methods and reasoning; understand applications to computer science
				Solve problems using graph theory.

9. Students will use spatial reasoning and apply the properties and relationships of geometric figures to represent, investigate, analyse, and solve problems.

Grade 6	Grade 7	Grade 8	Grade 9 & 10	Grade 11 & 12
Connect a graph or sketch with a real life scenario.	Describe the relationship between an equation and its graph.	Describe the relationship between an equation and its graph.	Be familiar with the graphs of the following equations and be able to apply them to problem solving: $y = \sin x$; $y = \cos x$; $y = \tan x$.	Be familiar with the graphs of trigonometric functions and be able to apply them to problem solving.
Classify and sketch a variety of two-dimensional regular and irregular figures.	Model, classify, compare, and sketch a variety of two- and three-dimensional regular and irregular figures.	Model, classify, compare, and sketch a variety of two- and three-dimensional regular and irregular figures.	Construct geometric models, transformations, and scale drawings using a variety of methods and tools (such as paper folding or protractor).	Understand two-dimensional representations of three-dimensional objects (e.g., draw a basic cube).
	Use coordinate geometry to represent and interpret relationships defined by equations and formulas (for example, distance, mid-point), translating among ordered pairs, graphs, and equations.	Apply properties of equality and proportionality to solve problems involving congruent or similar shapes.	Create two-dimensional representations of three-dimensional objects (e.g., draw a basic cube). Identify congruent and similar figures; apply this information to solve problems	Identify congruent and similar figures; apply this information to solve problems
Describe and apply geometric properties and relationships (such as parallelism and perpendicularity).	Describe and apply geometric properties and relationships (such as parallelism, perpendicularity).	Describe and apply geometric properties and relationships (such as congruence, perpendicularity).		
Describe and apply a variety of strategies for determining circumference, perimeter, area, surface area, angle measure.	Describe and apply a variety of strategies for determining circumference, perimeter, area, surface area, angle measure, and volume.	Describe and apply a variety of strategies for determining circumference, perimeter, area, surface area, angle measure, and volume.		
	Apply the Pythagorean theorem.	Explain and apply the Pythagorean theorem.	Use basic trigonometric ratios and trigonometric laws (triangle trigonometry) to solve problems involving indirect measurement.	Use basic trigonometric ratios and trigonometric laws (triangle trigonometry) to solve problems involving indirect measurement.
Draw and describe the results of transformations, including translations, rotations, and reflections.	Draw and describe the results of transformations, including rotations and reflections.	Draw and describe the results of transformations, including translations, rotations, reflections, and dilations (shrinking or enlarging), using proper notation.	Solve problems involving the laws of sines and cosines	Use trigonometric identities to solve equations.
			Apply the Pythagorean theorem and trigonometry to 3 dimensional problems.	Use radians and use formulae for arc length and sector area in both degrees and radians.
			Know properties of circles: apply circle theorems.	Use vector methods, matrices, and transformations.
			Use coordinate geometry to perform transformations	
			Understand the concept of a vector.	
			Use coordinate geometry to graph linear and quadratic equations, determine slopes of lines, identify parallel and perpendicular lines, and find possible solutions to sets of equations	Use coordinate geometry to graph linear and quadratic equations, determine slopes of lines, identify parallel and perpendicular lines, and find possible solutions to sets of equations.

10. Students will pose a question, collect, organize, analyse, and represent data in order to make decisions and predictions.

Grade 6	Grade 7	Grade 8	Grade 9 & 10	Grade 11&12
Formulate and solve problems by collecting, organizing, analysing (including comparing and contrasting), and displaying data in a variety of ways (including histograms, surveys, circle graphs) by hand and with technology.	Formulate and solve problems by collecting, organizing, analysing (including comparing and contrasting), and displaying data in a variety of ways (including histograms, surveys, circle graphs) by hand and with technology.	Formulate and solve problems by collecting, organizing, analysing (including comparing and contrasting), and displaying data in a variety of ways (including stem and leaf plots, histograms, whisker plots, surveys, circle graphs) by hand and with technology.	Formulate hypotheses; collect, organize, and represent data to prove/disprove those hypotheses.	Formulate hypotheses; collect, organize, and represent data to prove/disprove those hypotheses.
Find, describe, and interpret the arithmetic mean, median, and mode as measures of the centre of a data set; select the appropriate measure in particular situations to most accurately and fairly represent the data.	Find, describe, and interpret the arithmetic mean, median, and mode as measures of the centre of a data set; select the appropriate measure in particular situations to most accurately and fairly represent the data.	Find, describe, and interpret the arithmetic mean, median, and mode as measures of the centre of a data set.	Summarize and interpret single-variable data by choosing measures of central tendency and dispersion.	Summarize and interpret single-variable data by choosing measures of central tendency and dispersion.
	Describe ways to define a sample group; analyse a sample to make inferences about a population.		Analyse the validity of statistical conclusions.	Analyse the validity of statistical conclusions.
	Design and conduct a simulation to study a problem and communicate the results.			Describe the attributes of several common distributions (e.g., normal, uniform, Poisson, exponential), indicating data sets that would be expected to follow each type of distribution.

11. Students will understand and apply basic concepts of probability.

Grade 6	Grade 7	Grade 8	Grade 9 & 10	Grade 11 & 12
	Determine theoretical probability using a variety of methods, compare theoretical expectations to experimental results.	Determine theoretical probability using a variety of methods, including creating a sample space; compare theoretical expectations to experimental results.	Find the probability of simple events; compound events, and independent events.	Find the probability of simple events; compound events, and independent events using a variety of methods including the fundamental counting principle.
Design, conduct, and analyse the results of probability experiments.	Conduct, and analyse the results of probability experiments.	Design, conduct, and analyse the results of probability experiments.		Describe the normal curve in general terms and use its properties to answer questions about sets of data.